$$W = C\int_1^2 V^{-n}\,dV = \left[\frac{CV_2^{1-n} - CV_1^{1-n}}{1-n}\right]$$

$$C = p_2V_2^n = p_1V_1^n \tag{14}$$

Therefore, Eq. 14 can be expressed as:

$$W = \left[\frac{p_2V_2 - p_1V_1}{1-n}\right] \tag{15}$$

Using the ideal-gas laws:

$p_2V_2 = mRT_2$

and:

$p_1V_1 = mRT_1$

Eq. 15 is given by:

$$W = \frac{mR\,(T_2 - T_1)}{1-n} \tag{16}$$

For polytropic compression, work done is defined by:

$$W = \frac{nR(T_2 - T_1)}{1-n} \tag{17}$$

The polytropic compression work can be expressed as:

$$W = \frac{n}{n-1}[p_1V_1 - p_2V_2] \tag{18}$$

Since the process from 1 to 2 is polytropic, then:

$$p_1V_1^n = P_2V_2^n \tag{19}$$

and:

$$\frac{V_2}{V_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} \tag{20}$$

From Eq. 18:

$$W = \frac{n}{n-1}p_1V_1\left[1 - \frac{p_2V_2}{p_1V_1}\right] \tag{21}$$

and:

$$\frac{p_2V_2}{p_1V_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \tag{22}$$

Substituting Eq. 22 into Eq. 21:

$$W = \frac{n}{n-1}p_1V_1\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right] \tag{23}$$

where:

$R_c = \dfrac{p_2}{p_1}$ = compression ratio

Eq. 23 is given by:

$$W = \frac{n}{n-1}p_1V_1\left[1 - R_c^{\frac{n-1}{n}}\right] \tag{24}$$

Polytropic compressor. Real compression processes operate between adiabatic and isothermal compression. Actual compression processes are polytropic because the gas being compressed is not at constant entropy as in the adiabatic process, or at constant temperature as in the isothermal processes. Generally, compressors have performance characteristics analogous to pumps. Their performance curves relate flow capacity to head. The head developed by a fluid between states 1 and 2 can be derived from the general thermodynamic equation.

$$H = \int_{p_1}^{p_2} \overline{V}\,dP \tag{25}$$

where

H = head, kJ/kg
p = pressure, bara
$\overline{V}$ = specific volume of the fluid, m^3/kg

For a polytropic compression, the pressure-volume relationship is

$pV^n = constant$

or:

$$V = \frac{C_1}{p^{\frac{1}{n}}} \tag{26}$$

where:

V = mole volume, $m^3/(kg\ mol)$

For the polytropic head, H_p, V can be substituted in Eq. 25. The polytropic head is defined by:

$$H_p = \int_{p_1}^{p_2} \frac{C_1}{p^{\frac{1}{n}}}\,dP \tag{27}$$

by integrating Eq. 27, H_p becomes:

$$H_p = C_1\left(\frac{n}{n-1}\right)\left[p_2^{\frac{n-1}{n}} - p_1^{\frac{n-1}{n}}\right] \tag{28}$$

$$= C_1\left(\frac{n}{n-1}\right)p_1^{\left(\frac{n-1}{n}\right)}\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] \tag{29}$$

where:

$$p_1^{\frac{1}{n}}V_1 = p_2^{\frac{1}{n}}V_2 = C_1 \quad \text{and} \quad R_c = \frac{p_2}{p_1} \tag{30}$$

Substituting these into Eq. 29, to eliminate C_1 gives:

$$H_p = \left(\frac{n}{n-1}\right)p_1V_1\left[R_c^{\frac{n-1}{n}} - 1\right] \tag{31}$$

Using the gas law relationship: